NONLINEAR MODELING OF ANNUAL RUNOFF OF MAIN RIVERS IN BELARUS

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Abstract. The article investigates the problem of mathematical description of long-term fluctuations of river runoff, which is relevant for solving problems of modeling and forecasting in engineering hydrology. The description of the process of runoff fluctuations is based on the stochastic differential equations of Orshtein-Uhlenbeck and Fokker-Planck-Kolmogorov. A technique that makes it possible to apply low-parameter nonlinear dynamic models of river runoff has been developed. Mechanisms of the cyclicity of long-term fluctuations the Pripyat, Neman, West Dvina, Dnieper, and Berezina rivers are described. Comparison of the forecasting results according to the methodology developed by us showed better results than the modeling method using a simple Markov chain. The nonlinear model makes it possible to predict a series that has a correlation function similar to the original series with a shift of 4 or more years, and the Markov model gives good results only for an autocorrelation function with a shift of one year. The simulated series of annual runoff have statistical parameters that differ from the parameters of the original series within ±5-10%.

Keywords: stochastic hydrology, nonlinear modeling, river runoff, forecast.

Introduction

Recently, the problem of long-term fluctuations of river runoff has been widely studied. Runoff fluctuations have a pronounced cyclicality with possible positive or negative trends. Creation of deterministic models of runoff and artificial hydrological series makes it possible to solve predictive problems of engineering hydrology. For example, forecasting spring floods, natural disasters, taking into account current climatic fluctuations [1; 2]. In the second half of the last century research was carried out on the creation of deterministic models of the processes of river runoff formation using the methods of mathematical physics and their use in hydrological forecasts and calculations. However, the need for the development of new methods for the application of low-parameter nonlinear dynamic models of river runoff, which makes it possible to describe the physical mechanisms of the cyclicity of long-term fluctuations in river runoff, remains relevant [3-6]. The Soil and Water Assessment Tool (SWAT) is used to model water, sediment, and nutrient yield in a watershed using input data from GIS and the application of various farming, climate change, and land practices [7; 8]. The Soil and Water Integrated Model (SWIM) is used to study climate-induced runoff changes, extreme flood analysis, agricultural production, and so on [9; 10]. The use of artificial intelligence methods as an approach to solving complex nonlinear problems and predicting lake levels has increased [11; 12].

Materials and methods

Let $\overline{Q}$ be the average long-term water discharge, and Q_t – the water discharge at time *t*. Then, taking $X_t = (Q_t - \overline{Q})/\overline{Q}$, the process of long-term runoff fluctuations can be described using the following Orshtein-Uhlenbeck stochastic differential equation with continuous time t [13]

$$dX_t = -kX_t dt + \sigma dW_t, \tag{1}$$

where k^{-1} – relaxation time of river runoff;
σ – intensity of "white noise";
W_t – standard Wiener process.

The intensity of "white noise" is defined as $\sigma = C_v\sqrt{2k}$, where C_v is the coefficient of variation of the river runoff, and the coefficient k is from the following relation $k = -\ln r$, where r is the autocorrelation function of fluctuations in the river runoff.

Equation (1) corresponds to the Fokker-Planck-Kolmogorov equation

$$\frac{\partial p}{\partial t} = \frac{\partial}{\partial x}(kxp) + \frac{\sigma^2}{2}\frac{\partial^2 p}{\partial x^2}, \quad -\infty < x < \infty, \tag{2}$$

and the inverse equation has the form

$$\frac{\partial}{\partial t}p(x,t/y,0) = -ky\frac{\partial}{\partial y}p(x,t/y,0) + \frac{1}{2}\sigma^2\frac{\partial^2 p(x,t/y,0)}{\partial y^2}, \quad -\infty < x < \infty, \qquad (3)$$

since the random fluctuations of the runoff are homogeneous in time, that means that the relation $p(x,t/y,0) = p(x,0/y,t)$ holds.

Consider the following problem in stochastic hydrology. Let at the initial moment of time $t = 0$ the runoff is equal to Q, and Q_* is some fixed value of the runoff. It is required to determine the period of time during which the runoff value will be within $[Q_*,\infty)$. Let T be the moment in time when the runoff value leaves the half-interval $[Q_*,\infty)$.

Then

$$prob(T \geq t) = G(Q,t) \quad G(Q,t) = \int_{Q_*}^{\infty} p(x,t/y,0)dx.$$

Integrating (3) from Q_* to ∞ over x, we obtain

$$\frac{\partial G(Q,t)}{\partial t} = -kQ\frac{\partial G(Q,t)}{\partial Q} + \frac{\sigma^2}{2}\frac{\partial^2 G(Q,t)}{\partial Q^2}.$$

The boundary conditions are determined proceeding from the absorption of the value of the function at $Q = Q_*$, as well as from the reflection at infinity, that is

$$G(Q,t)|_{Q=Q_*} = 0, \quad \left.\frac{\partial G(Q,t)}{\partial Q}\right|_{Q=\infty} = 0.$$

The average time to reach the boundary Q_* is determined by the following relation

$$T_1 = -\int_0^{\infty} t\frac{\partial G(Q,t)}{\partial t}dt = \int_0^{\infty} G(Q,t)dt.$$

Integrating (3) over t from 0 to ∞ and taking into account that

$$\int_0^{\infty}\frac{\partial G}{\partial t}dt = G(x,\infty) - G(x,0) = -1,$$

we obtain the following equation for

$$T_1 1\frac{1}{2}\sigma^2\frac{d^2T_1}{dQ^2} - kQ\frac{dT_1}{dQ} = -1 \text{ at } \left.\frac{dT_1}{dQ}\right|_{Q=\infty} = 0, \; T_1(Q)|_{Q=Q_*} = 0.$$

Introducing dimensionless quantities

$$\theta_1 = kT_1, \quad \xi = Q\sqrt{\frac{2k}{\sigma^2}} = \frac{Q}{C_v}, \quad \xi_* = Q_*\sqrt{\frac{2k}{\sigma^2}} = \frac{Q_*}{C_v},$$

we get

$$\frac{d^2\theta_1}{d\xi^2} - \xi\frac{d\theta_1}{d\xi} = -1, \quad \left.\frac{d\theta_1}{d\xi}\right|_{\xi=\infty} = 0, \quad \theta_1(\xi)|_{\xi=\xi_*} = 0. \qquad (4)$$

Integrating system (4) by the numerical method [14; 15], we get the results that are given in Table 1.

Table 1

Solutions of equation (4)

ξ_*	ξ											
	-2.5	-2.0	-1.5	-1.0	-0.5	0	0.5	1.0	1.5	2.0	2.5	3.0
-3.0	59.9	76.5	82.3	84.8	86.1	86.9	87.5	87.8	88.1	88.4	88.6	88.7
-2.5	-	16.6	22,4	24.9	26.2	27.0	27.5	27.9	28.2	28.4	28.6	28.8
-2.0	-	-	5.8	8.3	9.6	10.4	10.9	11.3	11.6	11.8	12.0	12.2
-1.5	-	-	-	2.5	3.8	4.6	5.1	5.5	5.8	6.0	6.2	6.4
-1.0	-	-	-	-	1.3	2.1	2.6	3.0	3.3	3.5	3.7	3.9
-0.5	-	-	-	-	-	0.7	1.3	1.7	2.0	2.2	2.4	2.6
0	-	-	-	-	-	-	0.5	0.9	1.2	1.4	1.6	1.8

Now, at the initial moment of time $t = 0$, the runoff is equal to Q, and Q_* is also some fixed value of the runoff, but already greater than the initial one. To determine the period of time during which the runoff value will be within $(\infty, Q_*]$, system (4) with modified initial conditions is used, that is,

$$G(Q,t)|_{Q = Q_*} = 0, \quad \frac{\partial G(Q,t)}{\partial Q}\bigg|_{Q = -\infty} = 0.$$

Then we will have

$$\frac{d^2\theta_1}{d\xi^2} - \xi\frac{d\theta_1}{d\xi} = -1, \quad \frac{d\theta_1}{d\xi}\bigg|_{\xi = -\infty} = 0, \quad \theta_1(\xi)|_{\xi = \xi_*} = 0. \tag{5}$$

The solution to the system (5) gives the same results as given in the Table 1, only the values ξ and ξ_* are taken with opposite signs.

Results and discussion

The above methodology for determining runoff changes was proposed for 5 main rivers of Belarus [16; 17]. The data were used for the period of instrumental observations in the following sections: the Pripyat River at Mozyr, the Neman River at Grodno, the West Dvina River at Vitebsk, the Dnieper River at Mogilev, and the Berezina River at Bobruisk (Fig. 1).

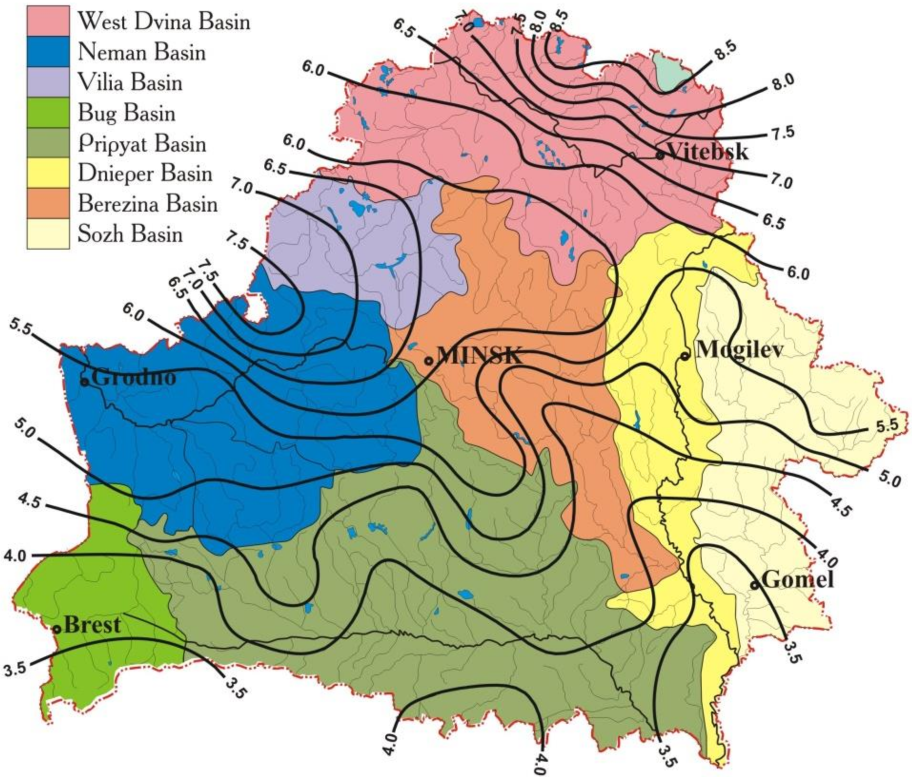

Fig. 1. **Map of the mean annual river runoff in Belarus (1956-2015), l·(s·km^2)$^{-1}$ [18]**

Let us consider as an example the calculation of the time of change in the values of the annual runoff of the Pripyat River at Mozyr. Table 2 shows the values of the main statistical parameters of the time series of the annual water discharge of the Pripyat River at Mozyr.

Table 2

Main statistical parameters of the time series of the annual water discharge of the Pripyat River at Mozyr

$\boldsymbol{Q_{mean}}$, m^3·s^{-1}	σ, m^3·s^{-1}	$\boldsymbol{C_v}$	$r(1)$
390	123	0.32	0.29

The coefficient k is determined by the formula $k = -\ln r = -\ln 0.29 = 1.24$.

Let at the initial moment of time $t = 0$ the runoff is $Q = 640$ m^3·s^{-1}, and the fixed value of the runoff is $Q_* = 200$ m^3·s^{-1}. Then ξ is defined as the deviation of the initial runoff value from the annual average one in fractions of C_v, i.e.

$$\xi = \frac{Q - Q_{mean}}{Q_{mean} \cdot C_v} = 2.$$

Similarly, ξ_* is defined as

$$\xi_* = \frac{Q_* - Q_{mean}}{Q_{mean} \cdot C_v} = -1.5.$$

According to the values of ξ and ξ_* from Table 1 we find the value $\theta_1 = 6.0$. Then the period of time during which the runoff value will be in the range of $[Q_*, \infty)$ is defined as the quotient θ_1 and k:

$$T_1 = \frac{\theta_1}{k} = 4.9.$$

The values of the dimensional time depending on the values of Q and $Q_*$are presented in Table 3.

Table 3

Values of the time of change in the annual runoff of the Pripyat River at Mozyr, years

Fixed runoff value, m^3·s^{-1}	Runoff at the initial moment of time, m^3·s^{-1}										
	200	250	300	350	400	450	500	550	600	650	700
200	0	1.8	2.8	3.5	3.9	4.3	4.5	4.7	4.9	5.0	5.2
250	0.2	0	1.0	1.7	2.1	2.5	2.7	2.9	3.1	3.2	3.4
300	0.4	0.2	0	0.7	1.1	1.4	1.7	1.9	2.1	2.2	2.3
350	0.7	0.5	0.3	0	0.4	0.8	1.0	1.2	1.4	1.5	1.7
400	1.0	0.9	0.7	0.4	0	0.3	0.6	0.8	0.9	1.1	1.2
450	1.6	1.4	1.2	0.9	0.5	0	0.3	0.5	0.6	0.8	0.9
500	2.4	2.1	1.9	1.7	1.3	0.8	0	0.2	0.4	0.5	0.6
550	3.6	3.4	3.2	2.9	2.6	2.0	1.3	0	0.2	0.3	0.4
600	5.9	5.7	5.5	5.2	4.8	4.3	3.6	2.3	0	0.1	0.3
650	10.7	10.5	10.3	10.0	9.7	9.1	8.4	7.1	4.8	0	0.1
700	22.0	21.8	21.6	21.4	21.0	20.5	19.7	18.4	16.1	11.3	0

Let us calculate the values of the dimensional time required for modeling the hydrological series for the remaining four sections. Table 4 shows the values of the main statistical parameters of the time series of annual water discharge for four sections.

Table 4

Main statistical parameters of the time series of the annual water discharge of the cross sections for the main rivers of Belarus

River - Section	Q_{mean}, m^3·s^{-1}	σ, m^3·s^{-1}	C_v	$r(1)$
Neman River at Grodno	197	35.5	0.18	0.16
West Dvina River at Vitebsk	226	61.6	0.27	0.31
Dnieper River at Mogilev	143	34.9	0.24	0.22
Berezina River at Bobruisk	119	22.9	0.19	0.05

Dimensional time values depending on Q and Q_* values are presented for sections of the Neman River at Grodno, the West Dvina River at Vitebsk, the Dnieper River at Mogilev, the Berezina River at Bobruisk in Tables 5-8.

Modeling of artificial hydrological series for sections of practically unlimited duration (more than 1000 years) is carried out using the results of Tables 3, 5-8, as well as using a simple Markov chain [19].

At the initial moment of time, a random value of the supply is simulated by means of a drawing, then the value of the discharge is determined according to the theoretical curve of supply predefined for the time series, which is plotted on the time scale.

The second modeled value of the supply makes it possible to determine the next value of the runoff. Having the values of runoff at the initial and final moments of time, according to Table 3, the nearest whole time interval is found, after which the second runoff value is plotted on the scale. Further, the next value of the supply is taken, according to which the value of the runoff is determined and the time interval plotted on the scale. Similarly, the simulation continues until the end of the time scale is reached, after which the process starts over, but when filling the scale, if the abscissa values coincide, priority is given to the value found at earlier stages of modeling. The first free value on the time scale is used to determine the runoff value at the previous moment in time, and from it – the discharge value at the current moment using a simple Markov chain [19]

$$x_{i+1} = 1 + r(x_i - 1) + F_{i+1} C_v \sqrt{1 - r^2}, \quad (6)$$

where x_{i+1} – value of the modular coefficient of the annual runoff volume in the (i + 1)-th year;
r – autocorrelation coefficient;
x_i – value of the modular coefficient of the annual runoff volume in the i -th year;
C_v – coefficient of variation.

The value of the function F is determined by the following formula

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{z^2}{2}} dz.$$

The obtained value of the annual runoff at the current moment is the initial one for subsequent modeling. The process continues until the entire scale is filled. The number of stages is limited and cannot exceed the duration of the modeled hydrological series.

Table 5

Values of the time of change in the annual runoff of the Neman River at Grodno, years

Fixed runoff value, m³·s⁻¹	**Runoff at the initial moment of time, m³·s⁻¹**										
	100	120	140	160	180	200	220	240	260	280	300
100	0	16.9	21.7	23.5	24.3	24.7	25.0	25.3	25.4	25.5	25.6
120	0.1	0	4.7	6.5	7.3	7.8	8.1	8.3	8.4	8.5	8.7
140	0.3	0.1	0	1.7	2.6	3.0	3.3	3.5	3.7	3.8	3.9
160	0.4	0.3	0.2	0	0.8	1.2	1.6	1.8	1.9	2.1	2.2
180	0.6	0.5	0.4	0.2	0	0.5	0.8	1.0	1.1	1.3	1.4
200	1.0	0.9	0.7	0.6	0.3	0	0.3	0.5	0.7	0.8	0.9
220	1.5	1.4	1.3	1.1	0.9	0.5	0	0.2	0.4	0.5	0.6
240	2.5	2.4	2.3	2.1	1.9	1.5	1.0	0	0.2	0.3	0.4
260	4.8	4.7	4.6	4.4	4.2	3.8	3.3	2.3	0	0.1	0.2
280	11.5	11.4	11.3	11.1	10.9	10.6	10.0	9.0	6.7	0	0.1
300	38.0	37.9	37.8	37.6	37.4	37.0	36.5	35.5	33.2	26.5	0

Table 6

Values of the time of change in the annual runoff of the West Dvina River at Vitebsk, years

Fixed runoff value, m³·s⁻¹	**Runoff at the initial moment of time, m³·s⁻¹**										
	100	125	150	175	200	225	250	275	300	325	350
100	0	5.1	7.5	8.8	9.6	10.2	10.6	10.9	11.1	11.3	11.4
125	0.2	0	2.4	3.8	4.6	5.1	5.5	5.8	6.0	6.2	6.4
150	0.3	0.2	0	1.3	2.1	2.7	3.1	3.4	3.6	3.8	4.0
175	0.6	0.4	0.2	0	0.8	1.4	1.8	2.0	2.3	2.5	2.6
200	0.8	0.7	0.5	0.3	0	0.5	0.9	1.2	1.5	1.6	1.8
225	1.2	1.0	0.9	0.7	0.4	0	0.4	0.7	0.9	1.1	1.3
250	1.7	1.6	1.4	1.2	0.9	0.5	0	0.3	0.5	0.7	0.9
275	2.5	2.3	2.1	1.9	1.6	1.3	0.8	0	0.2	0.4	0.6
300	3.7	3.5	3.4	3.1	2.9	2.5	2.0	1.2	0	0.2	0.4
325	5.9	5.7	5.6	5.3	5.1	4.7	4.2	3.4	2.2	0	0.2
350	10.4	10.2	10.0	9.8	9.5	9.1	8.6	7.9	6.6	4.5	0

Table 7

Values of the time of change in the annual runoff of the Dnieper River at Mogilev, years

Fixed runoff value, $m^3 \cdot s^{-1}$	Runoff at the initial moment of time, $m^3 \cdot s^{-1}$										
	80	95	110	125	140	155	170	185	200	215	230
80	0	2.6	3.9	4.6	5.1	5.4	5.7	5.9	6.0	6.2	6.3
95	0.1	0	1.3	2.0	2.5	2.8	3.1	3.3	3.4	3.6	3.7
110	0.3	0.2	0	0.7	1.2	1.5	1.8	2.0	2.1	2.3	2.4
125	0.5	0.4	0.2	0	0.5	0.8	1.0	1.2	1.4	1.5	1.6
140	0.8	0.7	0.5	0.3	0	0.3	0.6	0.8	0.9	1.0	1.1
155	1.2	1.1	0.9	0.7	1.4	0	0.2	0.4	0.6	0.7	0.8
170	1.8	1.7	1.5	1.3	1.0	0.6	0	0.2	0.3	0.5	0.6
185	2.9	2.7	2.6	2.3	2.0	1.6	1.0	0	0.1	0.3	0.4
200	4.8	4.7	4.5	4.3	4.0	3.6	3.0	1.9	0	0.1	0.2
215	9.0	8.9	8.7	8.5	8.2	7.8	7.2	6.2	4.2	0	0.1
230	20.0	19.9	19.7	19.5	19.2	18.8	18.2	17.2	15.2	11.0	0

Table 8

Values of the time of change in the annual runoff of the Berezina River at Bobruisk, years

Fixed runoff value, $m^3 \cdot s^{-1}$	Runoff at the initial moment of time, $m^3 \cdot s^{-1}$										
	80	90	100	110	120	130	140	150	160	170	180
80	0	1.1	1.7	20	2.2	24	2.5	2.6	2.7	2.7	2.8
90	0.1	0	0.6	0.9	1.1	1.3	1.4	1.5	1.6	1.6	1.7
100	0.2	0.1	0	0.3	0.6	0.7	0.8	0.9	1.0	1.0	1.1
110	0.3	0.2	0.1	0	0.2	0.4	0.5	0.6	0.7	0.7	0.8
120	0.5	0.4	0.3	0.2	0	0.2	0.3	0.4	0.4	0.5	0.5
130	0.7	0.6	0.5	0.4	0.2	0	0.1	0.2	0.3	0.3	0.4
140	1.1	1.0	0.9	0.8	0.6	0.4	0	0.1	0.2	0.2	0.3
150	1.7	1.6	1.5	1.4	1.3	1.0	0.6	0	0.1	0.1	0.2
160	3.0	2.9	2.8	2.7	2.5	2.3	1.9	1.3	0	0.1	0.1
170	6.0	5.9	5.8	5.7	5.5	5.3	4.9	4.3	3.0	0	0.1
180	14.4	14.3	14.2	14.1	13.9	13.7	13.4	12.7	11.4	8.4	0

Modeling the series of annual runoff by the indicated method gives acceptable results. The simulated series of annual water discharges using a simple Markov chain and a nonlinear method for the calculated sections have statistical parameters that differ from the parameters of the original series within ±5-10%. The values of the correlation functions for the original and modeled series for the Pripyat River at Mozyr are given in Table 9.

Table 9

Values of the statistical parameters of the initial and modeled time series of the annual water discharge of the Pripyat River at Mozyr

Time series	Statistical parameters							
	Q_{mean}, $m^3 \cdot s^{-1}$	σ, $m^3 \cdot s^{-1}$	C_v	$r(t–1)$	$r(t–4)$	$r(t–5)$	$r(t–10)$	$r(t–24)$
Initial	390	123	0.32	0.290	0.105	0.223	0.100	0.175
Modeled by simple Markov chain	388	117	0.30	0.303	0.005	-0.048	-0.036	0.035
Modeled by nonlinear method	399	137	0.34	-0.040	0.102	0.099	0.192	0.165

Using the apparatus of regression-correlation analysis, a complex Markov model was obtained with a shift of up to 50 years in annual fluctuations in water discharge, depending on $r(t – 1)$, $r(t – 4)$, $r(t – 5)$, $r(t – 10)$ and $r(t-24)$. The research has shown that modeling an artificial hydrological series by a simple Markov chain gives good results only for an autocorrelation function with a shift of one year, since this parameter is included in the modeling. A nonlinear model allows predicting a series with a correlation function similar to the initial series with a shift of 4 or more years.

Conclusions

The application of stochastic differential equations for the description and forecasting of long-term fluctuations in the annual runoff is proposed. The problem of stochastic hydrology of predicting the value of river runoff for 5 main rivers of Belarus is solved, and a method for modeling artificial hydrological series is proposed, which gives better results for predicting the "distant" correlation than the modeling method using a simple Markov chain. The simulated series of annual runoff have statistical parameters that differ from the parameters of the original series within ±5-10%. The research results can be applied in calculating and forecasting long-term fluctuations in river runoff of unexplored and poorly studied rivers in Belarus.

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